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Research Paper

Nonlinear Analysis of Two-Way Reinforced Concrete Slabs with Uncertainty Using the Random Variable Method

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Abstract

Reinforced two-way concrete slabs are essential structural components widely used in buildings and bridges due to their efficient load distribution. This study examines the effects of material property uncertainties, specifically concrete compressive strength and steel yield strength, using the random variable method. These key parameters are modeled as stochastic variables, and their influence on load-deflection response, fundamental frequency, and yield deformation is analyzed via finite element simulations in ABAQUS. Results show that variability in concrete and steel properties significantly impacts slab behavior, affecting serviceability and structural safety. The findings highlight the importance of considering material uncertainties in design for more reliable, optimized, and durable reinforced concrete structures capable of withstanding diverse loading conditions.

Keywords: Reinforced Two-Way Concrete Slabs; Yield Line Theory; Slab Structural Design; Stochastic Analysis; Monte Ca rlo Simulation

1. Introduction

Two-way concrete flat slabs are critical structural components frequently employed in construction to create horizontal surfaces capable of supporting loads over large areas without the necessity for beams or girders. These slabs are extensively utilized in various structures, including buildings, bridges, and pavements. A comprehensive understanding of their behavior under applied loads is essential for ensuring both safety and functionality. The design and analysis of two-way concrete flat slabs encompass a multitude of factors, including structural strength, load distribution, deflection control, and construction feasibility. Steel reinforcement, typically in the form of bars or mesh, is positioned within the concrete to resist bending and shear forces, thereby ensuring the slab's stability and overall strength [1]. The concept of utilizing reinforced concrete slabs to support larger areas without the need for beams or girders emerged in response to the increasing demand for more efficient and versatile structural solutions. Initially, one-way slabs predominated; however, engineers soon recognized the advantages of two-way slabs in effectively distributing loads in two directions. This realization facilitated their development and widespread application, particularly in constructions requiring expansive, open spaces, such as buildings and bridges.

The Finite Element Method (FEM) is a numerical technique employed to approximate solutions for partial differential equations, while Finite Element Analysis (FEA) is utilized to predict structural responses under varying conditions [2]. Given the inherent uncertainties associated with material properties, loading, and modeling, FEA frequently integrates reliability analysis, referred to as Stochastic Finite Element Analysis (SFEA). In SFEA, input parameters are treated as random variables, and methodologies such as Monte Carlo Simulation (MCS) are adopted to estimate statistical moments and probabilities of failure. The efficient integration of FEA software (e.g., ABAQUS) with reliability platforms presents challenges in minimizing computational trials and managing complex distributions of structural responses [3]. Ingerslev introduced the concept of "yield line theory," which evolved into a fundamental methodology for understanding the behavior of concrete slabs under loading conditions. By the 1940s, this method had been refined and applied to two-way slabs, providing engineers with a more systematic approach to the design of these structures. As a result, the acceptance of two-way slabs in construction increased, as they enabled a more efficient utilization of materials and improved load distribution. Further popularized in a modern context by Johansen in 1943, based on principles of plasticity, the yield line analysis method has been recognized as an effective technique for structural assessment. Kennedy and Goodchild [4] outlined various benefits associated with the application of yield line design, with particular emphasis on the cost-effective reinforcement configurations that it can produce. Consequently, the yield line method, recognized for its straightforwardness in comprehension and application, has emerged as a widely adopted technique in the analysis of structural elements. Although elastic analysis methods are typically utilized in practice, the implementation of yield line analysis can uncover additional reserve strength, thereby facilitating a more rational and cost-effective design of these slabs for heavy concentrated loads.

Nielsen [5] concentrated on the analysis and design of concrete structures, including two-way slabs, during the 1960s. His studies introduced advanced concepts pertinent to slab design. Nielsen examined the impacts of long-term loads (such as creep and shrinkage) on concrete slabs and proposed models to characterize the shear behavior of slabs subjected to loading. His research substantially influenced both the education and practical application of slab design. In the 1980s, Caltagirone [6] employed numerical methodologies, including the finite element method, to analyze concrete slabs. He investigated the behavior of these slabs under cyclic and dynamic loads. His work enhanced the precision of structural analysis and the simulation of concrete slab performance. Caltagirone also underscored the significance of concrete material compositions in determining slab strength.

The studies conducted by Fapohunda [7] and Vargas [8] addressed the objectives and requirements associated with the design of reinforced concrete structures. These investigations emphasized the application of the limit state analysis method, introducing and examining various types of limit states and their implementation in the design of reinforced concrete slabs. The significance of this method in the design of reinforced concrete members is particularly noteworthy due to the inherently higher level of uncertainty associated with concrete materials compared to other structural elements. The importance of limit state design, in conjunction with a probabilistic approach and the consideration of uncertainties in reinforced concrete structures, facilitates more accurate and comprehensive engineering assessments. In this context, Arafa [9] investigated reliability-based analysis for reinforced concrete beam sections under one of the limit states. He employed the Monte Carlo method to simulate the behavior of beam sections and conducted his analysis by calculating the reliability index at various levels of reinforcement.

This study aims to perform a stochastic analysis on slabs by accounting for the uncertainties associated with concrete and reinforcement properties. The slab design utilizes the yield line method, and the response analysis involves the development of ABAQUS software to analyze a square slab. Subsequently, stochastic analysis is conducted by introducing random variables, and the deflection of the slab is evaluated while considering the uncertainties in the aforementioned parameters.

2. Methodology

2.1 Yield Line Methodology

To analyze a rectangular reinforced concrete slab utilizing yield line theory, the following steps are generally undertaken:

1. Dividing the slab into a series of triangular or trapezoidal yield line regions. The number and placement of yield lines depend on the geometry and boundary conditions of the slab.
2. Assuming a yield line pattern or mechanism that satisfies the conditions of equilibrium and compatibility. This process entails selecting specific yield lines and determining their orientations. The choice of yield line pattern determines the geometry of the plastic mechanism and thus influences the internal moment arms and rotation paths. Consequently, it directly affects the internal work in the virtual work balance, altering the calculated ultimate load.
3. Calculating the ultimate load associated with the proposed yield mechanism. This is accomplished by evaluating the forces and moments acting along the yield lines and comparing these to the capacity of the concrete and reinforcing steel [10].
4. Verifying whether the calculated collapse load exceeds the applied load is essential for assessing the validity of the assumed yield mechanism. If it does not, the yield mechanism must be revised until a valid configuration is established [11].

The yield line method can be integrated with the principle of virtual work to examine the ultimate behavior of reinforced concrete slabs. This integrated approach is referred to as the yield line method with virtual work. In this methodology, the principles of virtual work are employed to ascertain the critical yield line pattern and the associated collapse load of a reinforced concrete slab. The principle of virtual work states that the work performed by external forces acting on a structure is equivalent to the work performed by internal forces in response to virtual displacements [1, 12]. The yield line method was employed as a preliminary design approach to establish an approximate steel reinforcement configuration, including bar size and spacing, for the ultimate load (wu) per reference codes. To obtain a more accurate evaluation of the reinforced concrete slab's behavior under loading and to consider nonlinear phenomena such as cracking, material plasticity, and stiffness degradation, a comprehensive finite element model was developed using ABAQUS. This simulation provided a precise representation of structural performance, validating the design assumptions under the applied load.

By integrating yield line theory with principles of virtual work, engineers can ascertain the critical yield line pattern and evaluate the collapse load of reinforced concrete slabs. This methodology considers the equilibrium and compatibility conditions of the structure while accounting for the virtual displacements and work performed by both internal and external forces. It is essential to acknowledge that the yield line method, in conjunction with virtual work, presupposes linear elastic behavior of the structures and disregards nonlinear effects, including cracking and plasticity [13].

Collectively, this approach may be limited in its capacity to accurately predict the behavior of structures exhibiting significant nonlinearities. In such instances, more sophisticated analysis techniques, such as nonlinear FEA, may be necessary. The virtual work method in yield line analysis is grounded in the principle that "when a rigid body, in equilibrium under a system of forces, undergoes a virtual displacement, the total virtual work done by the forces is equal to zero." The work performed by external forces (W_E), resulting from loads applied to the slab, must be equivalent to the work conducted by internal forces (W_I) along the yield lines, with no energy dissipation, as stated by Jackson [14]. For a square simply supported slab of dimension (l), as depicted in **Fig. 1**, the outcomes of internal and external virtual work are as follows:

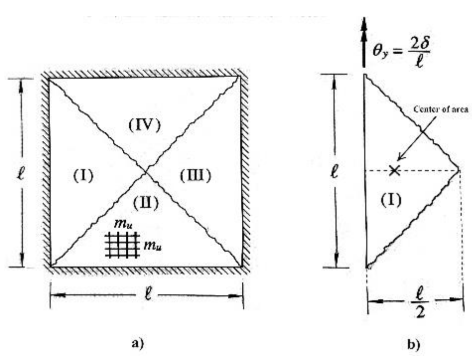

Fig. 1. a) Simply supported square slab [15] b) Rotation of the slab sections [15]

$$W_E = \frac{1}{12} w_u l^2 \delta \tag{1}$$

$$W_I = \Sigma (m_{ux} l_y \theta_y + m_{uy} l_x \theta_x) \tag{2}$$

$$W_I = 2 m_u \delta \tag{3}$$

$$W_E = W_I \tag{4}$$

$$m_u = \frac{w_u l^2}{24} \tag{5}$$

Where (w_u) represents the uniformly distributed load acting on the slab, and δ denotes displacement at the center of the slab. The flexural capacity of the slab parallel to the potential crack per unit width of the slab is denoted as (m_{ux} and m_{uy}). If there is a potential crack along a yield line at an angle α concerning the y-axis, then the rotation components of the slab section around the yield line are denoted as $\theta_x=\theta\sin\alpha$ and $\theta_y=\theta\cos\alpha$, while the projections of the crack along the x and y axes are $l_x= l\cos\alpha$ and $l_y= l\sin\alpha$, respectively. According to Eq. 4, representing the principle of virtual work, the external work done by the w_u over the δ equals the W_I performed by the bending moments m_{ux} *and* m_{uy} and the corresponding rotations θ_x and θ_y along the yield lines. To calculate the required bending reinforcement (steel) in the middle strip of the slab, the well-known Eq. (6) is utilized, Where (R_n) is nominal strength, (φ) is a strength reduction factor, (b) is the width of compression face of the slab and (d) is a distance from extreme compression fiber to centroid of longitudinal tension reinforcement. This equation provides a formulation for determining the necessary amount of steel reinforcement in the middle strip of the slab. [15-17].

$$R_n = \frac{M_u}{\phi b d^2} \tag{6}$$

The yield line method, when integrated with the principle of virtual work, serves as an efficient analytical tool for estimating the collapse load by identifying critical yield line patterns. However, this approach inherently depends on assumptions of idealized rigid-plastic behavior and linear-elastic deformation before collapse. Such simplifications fail to account for progressive cracking, tensile softening, and reinforcement yielding, which may significantly influence actual structural performance. To address these limitations, the slab was initially designed using the yield line method and subsequently modeled in ABAQUS to accurately simulate its performance under loading conditions. The finite element model facilitated the incorporation of nonlinear material properties and boundary effects, thereby enhancing the accuracy of structural response predictions and validating the assumptions underlying the yield line-based design. This comprehensive modeling approach ensured that the assumed collapse mechanism and moment capacities were aligned with the slab's actual response to loading.

2.2. Random Variable Method

The random variable method constitutes a mathematical technique employed to analyze and address problems associated with random variables. A random variable is defined as a variable whose value depends on the outcome of a stochastic event. The method entails the formulation of the random variable, the identification of its probability distribution, the computation of statistics such as the mean and variance, the application of mathematical operations and techniques, and the interpretation of the resulting outcomes [18]. This methodology finds applications in probability theory, statistics, and various other disciplines, facilitating the modeling and analysis of uncertain events. The random variable method offers a systematic framework for comprehending and quantifying randomness and uncertainty.

2.3. Monte Carlo Simulation

MCS is a computational methodology employed to estimate and analyze the outcomes of complex systems or processes characterized by inherent uncertainty. This technique entails the generation of random samples for uncertain variables, simulating the system utilizing these samples, aggregating the results, and subsequently analyzing them to derive insights and inform decision-making. MCS is particularly advantageous in scenarios where analytical solutions are impractical or when addressing complex systems. It facilitates a probabilistic analysis that encompasses the complete spectrum of potential outcomes and finds robust applications in various fields, including finance, engineering, and risk analysis. [18, 19].

To implement MCS, the random variables of interest must be updated for each FEA trial. This update process is facilitated through the use of ABAQUS software, which is employed to construct a deterministic finite element model [20]. The concept of parameter updating is applied to revise the input random variables within the analysis. The initial step involves creating a finite element model using ABAQUS. Upon completion of the modeling process, random numbers are generated in MATLAB based on the mean and standard deviation (Std.) of the parameters. These generated numbers are subsequently input into ABAQUS for simulation. The resulting data are presented in the form of force-displacement curves.

The limit state function is defined mathematically as Eq.7:

$$g(x) = R - S \tag{7}$$

where R denotes the structural resistance and S represents the load effect. Failure occurs when g(X)≤0, meaning the applied load exceeds the resistance.

3. Verification of the Model

Accurate modeling and analysis of RC slabs are critical for ensuring structural safety under various loading conditions and require robust validation methods to confirm predictive accuracy. Verification entails comparing model outputs against benchmark data, typically derived from experimental results, to establish confidence in stochastic simulations that account for uncertainties, particularly in material properties and environmental factors. This study employs advanced statistical methodologies to validate the concrete slab model, thereby affirming its applicability for practical engineering applications.

The behavior of concrete in the second model under compression is characterized by utilizing the widely recognized Hognestad model, which provides a parabolic stress-strain relationship to represent the nonlinear behavior of concrete subjected to compressive forces. This model adeptly captures the initial linear elastic region, followed by nonlinear hardening and softening phases leading up to the ultimate strain. Building upon this framework, the Concrete Damage Plasticity (CDP) model is employed to simulate the complex behavior of concrete under combined tensile and compressive stresses [21]. The CDP model integrates both damage and plasticity mechanisms to account for the progressive degradation of concrete stiffness and the accumulation of irreversible deformations. It employs a damage parameter to represent microcracking in tension and crushing in compression, thereby providing a more accurate depiction of concrete's nonlinear response. In the context of reinforcing steel, Grade 400 reinforcing steel, characterized by a yield strength of 400 MPa, is utilized due to its advantageous blend of strength and ductility, which is essential for reinforced concrete applications. The steel reinforcement is represented through truss elements within an embedded region framework, facilitating a precise depiction of the reinforcement's behavior within the concrete matrix.

In the embedded region model, a perfect bond is assumed between the truss steel elements and the surrounding concrete, ensuring identical strains at their interface. In the embedded region model, it is assumed that the bond between the truss elements and concrete is perfect, meaning no relative slip occurs between the rebar and the concrete. Within the CDP framework, the post-cracking tensile behavior of concrete is modeled through tension stiffening, representing the gradual reduction of tensile capacity after cracking. This behavior is calibrated using experimental data and empirical relations, and is implemented in the model via a tensile stress–strain table. The tensile damage parameter increases progressively from no damage at initial strains to complete damage at higher cracking strains, reflecting the gradual evolution of microcracking. These assumptions are consistent with empirical observations and established design recommendations.

The compressive behavior of concrete follows the Hognestad stress-strain curve, as shown in **Fig. 2**, which includes three distinct phases. Initially, in the linear elastic phase, concrete exhibits a proportional stress-strain relationship governed by its modulus of elasticity until internal cracking begins. As the load increases, concrete enters the hardening phase, where the curve becomes nonlinear, stress continues to rise, and microcracks start forming, leading to stress concentration. Upon reaching the ultimate compressive strength, concrete transitions into the post-peak softening phase, where stress gradually decreases with increasing strain until complete failure occurs. Similarly, the tensile behavior of concrete, as illustrated in **Fig. 3**, consists of two main phases. In the linear elastic phase, stress increases proportionally with strain until reaching the ultimate tensile strength. Beyond this point, concrete enters the tensile softening phase, where cracks propagate, stress reduces, and the material loses continuity, ultimately leading to fracture [1, 22]. In the CDP model in ABAQUS, tensile stiffening is represented by defining the post-cracking behavior of concrete using stress-strain curves or damage parameters. It is assumed that concrete retains some tensile capacity after cracking, which gradually decreases with increasing strain. The tensile behavior after cracking is modeled based on the stress-strain curve, including a linear-elastic response up to the peak tensile strength f_{t0}, followed by nonlinear softening. This behavior is calibrated with experimental data and standard relations to accurately simulate the reinforced concrete response. The tensile damage parameter d_t is defined as increasing from zero (no damage) at initial strains to one (complete damage) at a specified cracking strain, reflecting the gradual evolution of tensile damage. The d_t values are extracted from the damage curve and input into the CDP model as a table [13, 23].

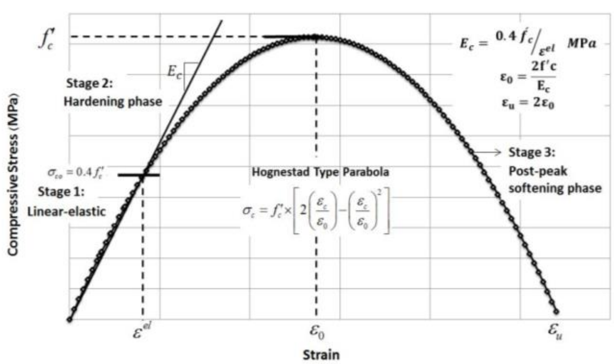

Fig. 2. Compressive Stress-Strain Curve of Concrete [23]

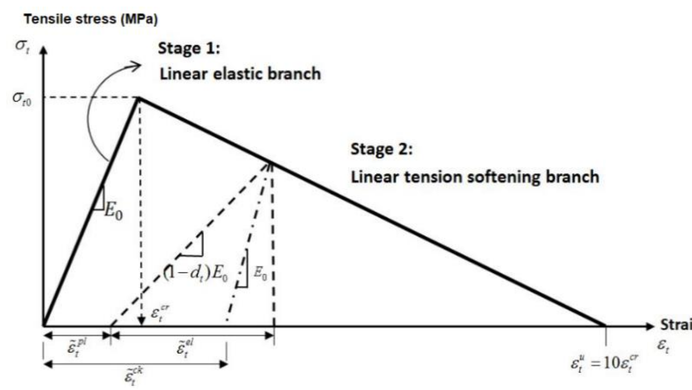

Fig. 3. Tensile Stress-Strain Curve of Concrete [23]

Jofriet and McNeice [24] conducted a validation test on a two-way reinforced concrete slab. The slab was supported at the corners and subjected to a central point load. The reinforcement ratio in both the x and y directions of the slab was identical; however, reinforcement was present solely in the tension face. The properties of the slab are detailed in **Fig. 4** and **Table 1** [24, 25]. Initially, in this study, the slab was modeled with dimensions identical to those of the experimental specimen. The mechanical properties of concrete and reinforcement were defined based on laboratory data. The mesh size and element type were selected to balance computational efficiency and accuracy. Boundary conditions replicated the test setup, with one pinned and three roller supports.

During preliminary modeling, failure was observed to initiate at the supports, contrary to the experimental results, in which cracking began at the slab center. To address this, four rigid plates were added at the support locations. Furthermore, a central rigid plate was introduced to prevent artificial stiffness effects at the load application point and to ensure realistic stress distribution. The uniform load was applied to this central plate and transferred to the slab, resulting in a simulated response that closely matched the experimental behavior, with failure initiating at the slab center as expected.

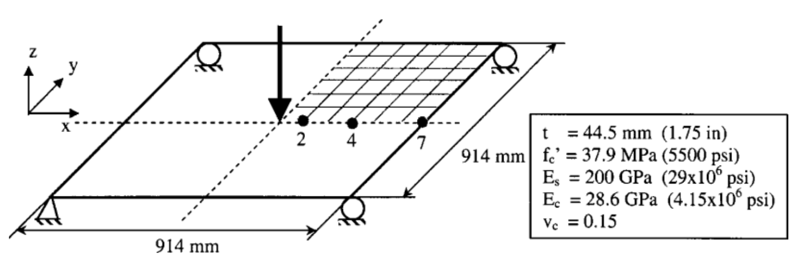

Fig. 4. Schematic of corner-supported reinforced concrete slab tested by Jofriet and McNeice [24]

Table 1. Properties of the specimen

Specimen	Dimensions	Concrete		Reinforcement	
		f_c' (MPa)	$E(MPa)$	P_x^* (%)	P_y^* (%)
McNeice's slab	914 * 914 * 44.50 (mm)	37.90	28.62	0.85	0.85

*Reinforcement ratio per layer

Figs. 5–7 depict the finite element mesh, reinforcement arrangement, and crack patterns of the slab model. A structured mesh composed of hexahedral elements was employed to achieve precise stress distribution. The reinforcement was represented using embedded truss elements, while crack patterns were visualized via plastic equivalent strain, emphasizing regions with notable tensile cracking.

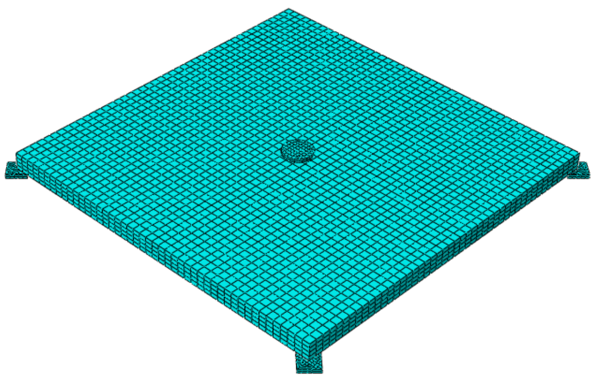

Fig. 5. Mesh pattern of McNeice's slab

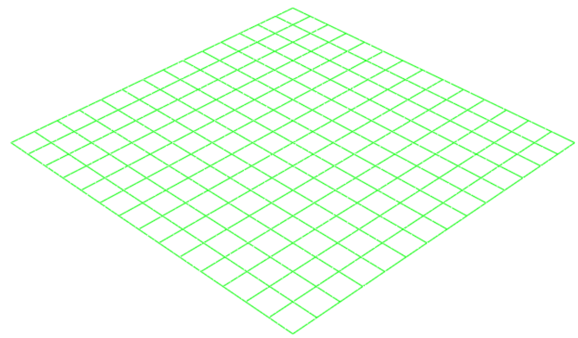

Fig. 6. Reinforcement layout

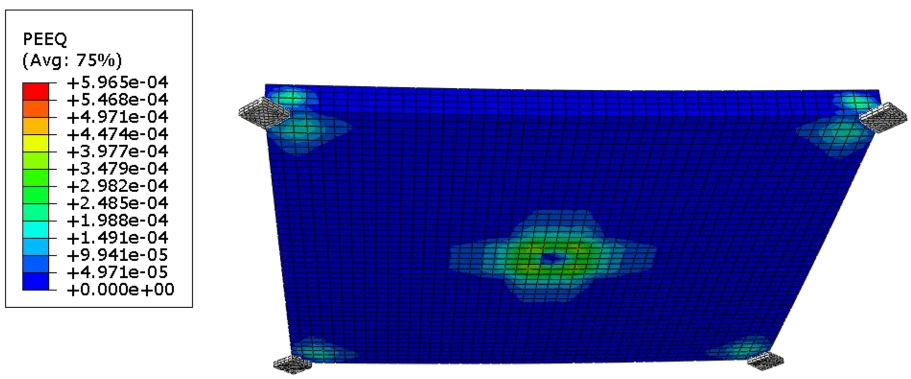

Fig. 7. Plastic equivalent strain

The graphs presented in **Fig. 8** illustrate a comparative analysis between the experimental results of a specific slab model and its counterpart simulated using ABAQUS software. The data depicted in the graphs indicate that the simulated model accurately reflects the behavior of the experimental model. Furthermore, the graphs substantiate the assertion that the simulated model has been effectively constructed and is in alignment with the experimental data.

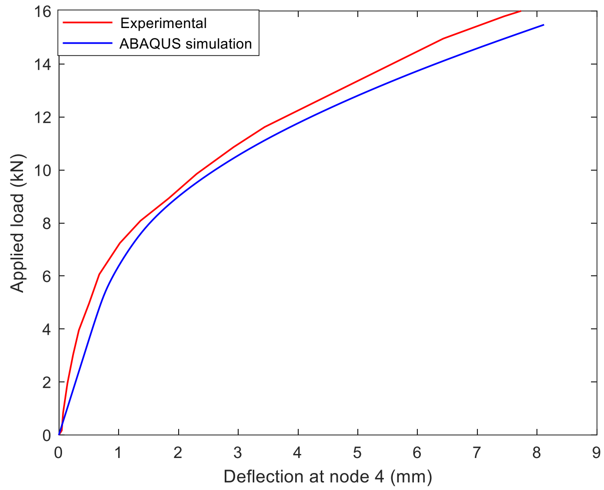

Fig. 8. Load-deflection at node 4 in Fig. 4

Table 2 provides a comparison of experimental and numerical outcomes for McNeice's slab, focusing on ultimate load and peak deflection. The results demonstrate a close correlation between the two approaches, with minor percentage deviations observed in both strength and displacement values.

Table 2. Comparison of experimental and numerical results for McNeice's slab

Test case	Exp. ultimate load (kN)	Num. ultimate load (kN)	Diff. (%)	Exp. peak deflection	Num. peak deflection	Diff. (%)
McNeice' slab	16	15.38	3.88	7.77	8.14	4.55

The displacement pattern obtained from the simulation, as illustrated in **Figs. 9** and **10** closely align with the displacement pattern presented in McNeice's thesis. This concordance suggests the accuracy and validity of the modeling undertaken, as well as its consistency with experimental results.

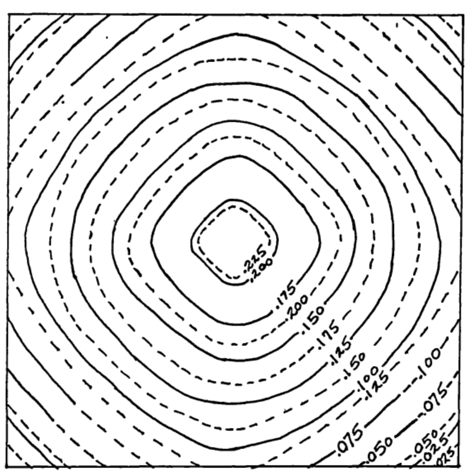

Fig. 9. The displacement pattern developed in McNeice's experimental slab

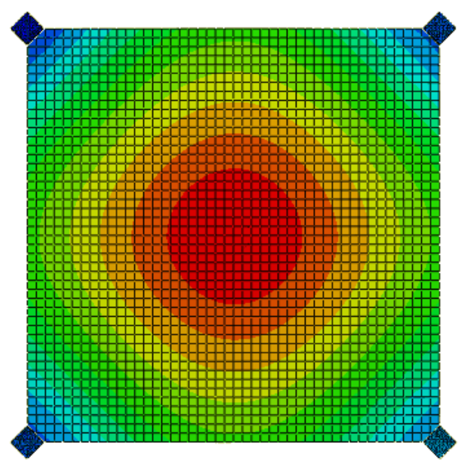

Fig. 10. The displacement pattern developed in ABAQUS

4. Sensitivity Analysis

To perform a local sensitivity analysis, an initial assessment was carried out by applying a ±10% coefficient of variation around the mean values of f'c and fy. As shown in **Fig. 11**, variations in these parameters affect the structural response, with fc demonstrating a more significant influence.

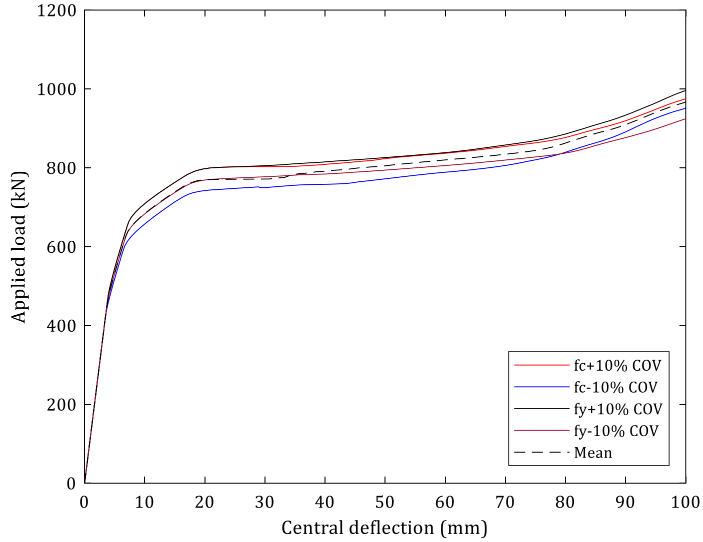

Fig.11. Sensitivity analysis

5. Stochastic Analyziz of Slab

Stochastic analysis of slabs entails the examination of their behavior under various uncertain factors, including material properties, loads, and boundary conditions. This approach employs random variables characterized by known probability distributions to evaluate the performance of the slab. MCS is frequently utilized to generate random samples and assess the behavior of the slab [26, 27]. In this context, a normal Probability density function (PDF) is considered for the input data, and an MCS comprising 500 iterations has been implemented for both models introduced in the ABAQUS software. Stochastic analysis yields valuable insights into the probabilistic behavior of structures, facilitating the estimation of failure probabilities and other performance metrics. This analysis is instrumental in the domains of design, optimization, risk management, and maintenance decision-making. Results are typically reported in the form of probability distributions or statistical moments. Accurate modeling of uncertainties and the judicious selection of probability distributions are essential for effective analysis.

5.1. Stochastic Analysis Framework Integrating MATLAB and ABAQUS

A deterministic finite element model of the reinforced concrete slab was constructed in ABAQUS with parametric representation of material properties. To incorporate uncertainties inherent in the materials, stochastic samples of concrete compressive strength and steel yield strength were generated in MATLAB according to normal distributions characterized by the mean and coefficient of variation values listed in Table 3. These sampled parameters were subsequently imported into ABAQUS input files to execute 500 MCS runs. For each simulation instance, the FE model was updated accordingly and analyzed, with critical outputs such as load-displacement responses and nodal displacements extracted. The simulation results were then imported back into MATLAB, where statistical evaluations, including calculation of mean values, standard deviations, coefficients of variation, and probability density functions, were conducted. This coupled computational approach facilitated efficient uncertainty propagation and provided a rigorous assessment of the slab's structural behavior under stochastic material variability.

6. Illustrative Example

This example focuses on the analysis of a square slab that is supported on all four sides and possesses the capacity to resist moments in all directions. The specific scenario under consideration involves a two-way slab with simple supports on each side, which is capable of withstanding moments in both directions. The slab is subjected to a uniformly distributed load along its span. The objective is to evaluate a proposed methodology that integrates deterministic and stochastic analyses to address uncertainties and assess failure probabilities. **Table 2** presents the details and characteristics of the illustrative example, along with a visual representation of the sequence of steps undertaken in this research. The deterministic FEA simulation results exhibit a reasonable correlation with the behavior of the tested specimens regarding ultimate load, deflection, and crack propagation [28]. In the probabilistic analysis, only the uncertainties associated with the material properties are considered.

Additionally, design codes such as ACI 318 [17]for punching shear, as well as the critical shear crack theory, are examined while taking into account the same input uncertainties. The results of this investigation offer valuable insights into the predictive capability of the proposed probabilistic FEA approach, which may be applied in future studies [29, 30].

Table 2. Properties of the illustrative example

Dimensions	Concrete		Reinforcement	
6000 * 6000 *200 (*mm*)	f_c' (*MPa*)	*E(MPa)*	P_x^* (%)	P_y^* (%)
	20.00	28.62	0.85	0.85

The stochastic input parameters, along with their mean Std. values, are presented in **Table 3** [31]. **Figs. 12** and **13** illustrate the probability density function (PDF) of the normal distribution for the yield strength of steel and the compressive strength of concrete, respectively.

Table 3. The stochastic input parameters

Parameters	Unit	Mean	*COV (%)	Std.	Reference
f_c'	*MPa*	27.8	10.7	3.0	[29]
f_y	*MPa*	496.0	9.0	44.8	[30]

*COV: Coefficient Of Variation

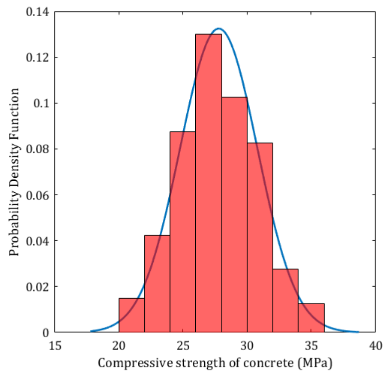

Fig. 12. The PDF of the yield strength of steel

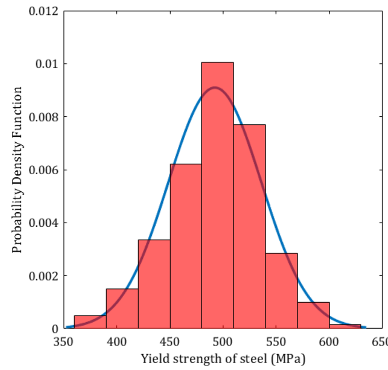

Fig. 13. The PDF of compressive strength of concrete

7. Results and Discussion

The compressive strength of concrete exerts a direct and significant influence on slab deflection. As the compressive strength of concrete increases, the elastic modulus and, consequently, the flexural rigidity of the slab improve, given that concrete serves as the primary load-bearing element under compression. This enhanced rigidity results in reduced displacement or deflection under applied loads, thereby decreasing the slab's flexibility and promoting more rigid structural behavior. This effect is particularly evident under service loads and within the elastic regime of concrete, wherein compressive strength plays a critical role in controlling deformation.

Conversely, the yield strength of steel (f_y) does not significantly affect slab deflection up to the yield load of the slab. At this stage, the stiffness of the slab is predominantly determined by the concrete properties, including its elastic modulus and compressive strength, while steel reinforcement primarily contributes to resisting tensile stresses without directly influencing deflection. This behavior persists as long as the stress in the reinforcement remains below the yield level, which typically corresponds to service loads or the initial loading phase.

Beyond the yield point, the scenario shifts. Once the reinforcement yields, the flexural stiffness of the slab decreases markedly, as steel can no longer elastically accommodate additional stress and enters the plastic range. In this condition, a higher yield strength (greater f_y) enables the slab to sustain greater loads, thereby increasing its load-carrying capacity. However, this enhanced capacity is accompanied by a significant increase in deflection. This phenomenon occurs because extensive yielding of the reinforcement leads to plastic deformation, which substantially amplifies deflection. This behavior reflects a trade-off between improved resistance and reduced control over deformation following yielding [32].

Following the analyses conducted using the CDP model, the modifications made to the slab and the reinforcement mesh were thoroughly examined. The results indicated that the changes in stress distribution and the behavior of the slab under load were significantly influenced by the CDP model. Specifically, as illustrated in **Figs. 14** and **15**, the progression of damage within the slab and the variations in stresses throughout the reinforcement mesh are distinctly observable. These findings underscore the efficacy of the CDP model in providing a more accurate simulation of the behavior of reinforced concrete slabs under diverse loading conditions. Notably, it was observed that, with increasing load, stresses became concentrated in specific regions of the reinforcement mesh, leading to marked alterations in the slab's performance. These analyses ultimately contribute to enhancing the precision of design and evaluation processes for reinforced concrete structures. In general, the compressive strength of concrete plays a pivotal role in controlling slab deflection. However, the yield strength of steel is vital for increasing load-carrying capacity and enhancing performance against bending loads.

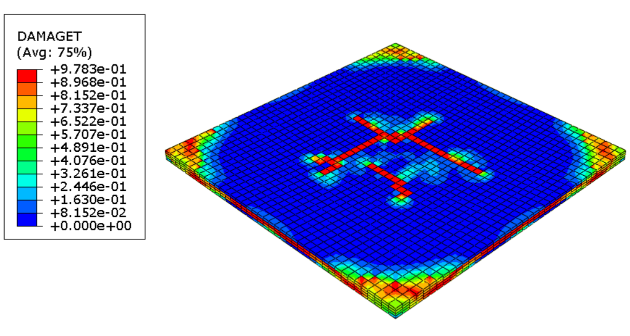

Fig. 14. Representation of damage progression in the CDP model

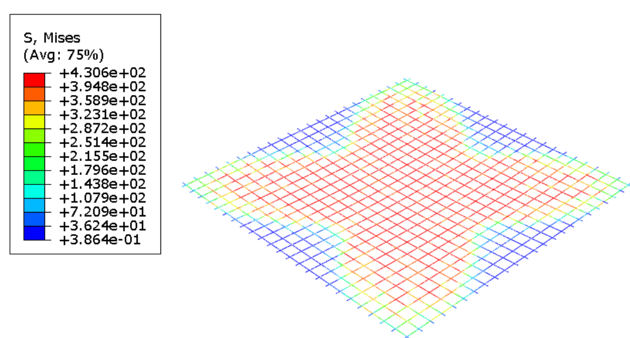

Fig. 15. Distribution of stresses in the reinforcement mesh

The analysis results, as shown in **Fig16**, reveal a distinct trend in the COV of slab deflection: it increases with load at initial to moderate levels, then decreases at higher loads. This behavior reflects an initial amplification of uncertainty followed by a reduction. At lower loads, the slab operates within the elastic or cracking regime, where minor fluctuations in material properties significantly affect stiffness and ‹thus‹deflection. The CDP model in ABAQUS captures this sensitivity, amplifying output variability. As loading increases, the stress distribution near critical zones worsens the impact of material uncertainties. However, once the slab enters full plasticity characterized by yielding and concrete crushing, its response becomes dominated by structural saturation. At this stage, the influence of material variability diminishes, leading to convergence in behavior across samples and a corresponding drop in COV. This trend highlights the interplay between nonlinear material behavior and the modeling capabilities of the CDP framework in simulating progressive damage and uncertainty propagation.

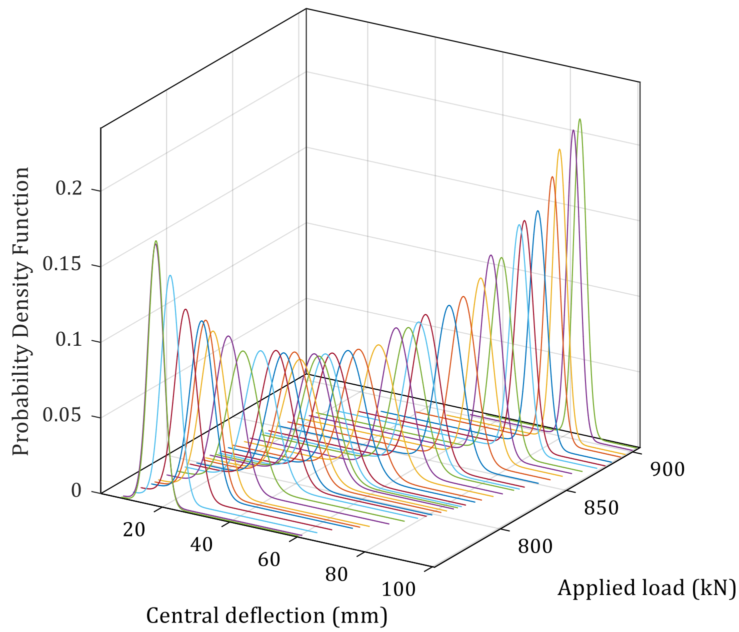

Fig. 16. Three-dimensional correlation of central deflection and applied load with COV for 6×6m RC slab

To present the results and analyses in an organized and comprehensible manner, a vertical normal distribution chart has been developed, which comprehensively and transparently displays the analysis results alongside the force-displacement distribution. In this chart, the distributions of the force-displacement curves and the final displacements are constrained, with these constraints observable in the normal distribution of the curves. For each chart, vertical normal distributions have been constructed, allowing for distinct observation of variations in force under three different conditions: average force, minimum force, and maximum force. These values are precisely identifiable in each chart. The average force represents the variation at the point of highest data concentration, while the minimum and maximum force values indicate the points at which the force variations reach their lowest and highest levels, respectively. This normal distribution demonstrates the trend in force variations and the system's behavior under varying conditions. **Fig. 17** illustrates the vertical probability density charts.

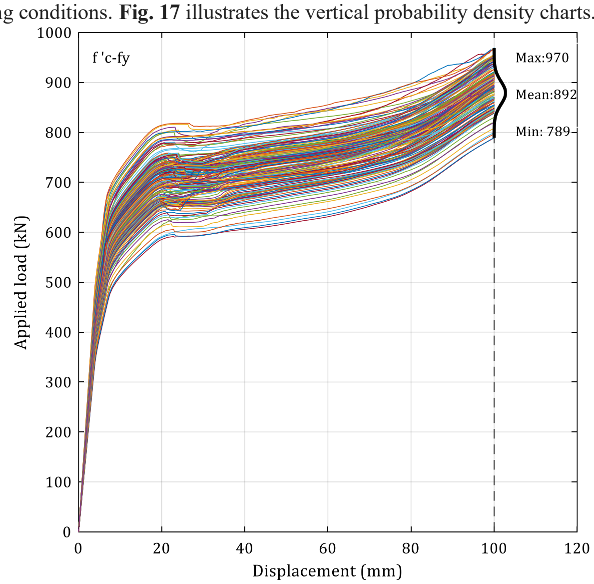

Fig. 17. Vertical PDF of ultimate strength

Based on Table 4, in the design load range, we observe that the correlations are quite low. This is expected and consistent with the fact that we are operating within the elastic range, where the current analysis approach is valid. Although at higher load levels these correlation values may increase and approach one, such high loads are rare for the slab to experience in practice. Therefore, at this stage, we are indeed within the design limit state. In our next step, we plan to perform a more comprehensive analysis to better calibrate and refine the parameters.

Table 4. Correlation analysis between input and output variables

Variable	Corr, f'c-deflection	Corr, f'c-load	Corr, fy-deflection	Corr, fy-load
design load	0.0093	0.0208	-0.0345	-0.0488

8. Conclusion

This investigation evaluates the variability of the ultimate load and yield pattern by incorporating uncertainties in the geometric properties and the material characteristics of concrete and steel in reinforced concrete slabs. Uncertainty analysis represents a highly effective approach for the design and assessment of structures, which has witnessed significant growth and advancement in recent years. It is recommended that reliability analysis be incorporated as an additional component in current standards, alongside the codification of bylaws and guidelines for loading and design.

This study involves a stochastic analysis of a reinforced concrete slab, with a specific emphasis on the variability of material properties, employing the MCS method. Generally, the compressive strength of concrete significantly influences slab deflection. However, the analysis results reveal that the COV of slab deflection initially increases with increasing load and subsequently decreases at higher load levels. This trend illustrates an initial amplification of uncertainty attributable to sensitivity to material properties, followed by a reduction as the slab transitions into the fully plastic stage. At this juncture, the structural response becomes saturated, leading to a diminished impact of material variability.

Author Contributions

Author 1 performed the primary experimental work, data collection, and analysis under the guidance and supervision of Authors 2 and 3. Authors 2 and 3 also contributed significantly to the stochastic modeling, theoretical support, and overall project supervision. The manuscript was drafted by Author 1 and subsequently reviewed, edited, and critically revised by Authors 2 and 3. All authors read, discussed, and approved the final manuscript.

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Conflict of Interest

The authors declare that they have no potential conflicts of interest related to the research, authorship, or publication of this article.

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Data Availability Statement

The datasets generated and/or analyzed during the current study are available from the corresponding author upon reasonable request.

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