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Research Paper

Steady State Moisture Diffusion Through Thickness of Plates, Cylindrical and Spherical Shells in Presence of Non-uniform Temperature

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Abstract

Determining temperature and moisture distribution is essential for hygrothermal analysis of structures. In most cases, the rate of moisture diffusion depends on temperature. Here, the Arrhenius law is used to investigate steady-state moisture diffusion through the thickness of plates, cylindrical, and spherical shells under non-isothermal conditions. The derived one-dimensional Fick's diffusion equation with a variable diffusion coefficient is solved analytically to determine the moisture profile through the thickness of structures. Temperature and moisture values prescribed on the inner and outer surfaces of the structures are used as boundary conditions to solve the equations. The effects of the surface temperatures on moisture distribution are then investigated. The steady-state moisture profile depends on the temperature boundary conditions, unlike in the case of uniform temperature distribution. The results showed that moisture concentration and content increased significantly when the dry surface was cooler than the moist surface, compared to the uniform temperature case. When thermal and moisture diffusion occur in opposite directions, the trend reverses, and both moisture content and concentration decrease.

Keywords: Fick diffusion; Non-Isothermal; Plate; Cylinder; Sphere.

1. Introduction

Plates, cylindrical, and spherical shells are key structural components widely used across engineering fields like civil, mechanical, aerospace, and marine engineering. In many of these applications, the structure is exposed to environmental conditions such as sunlight, rain, snow, or wind, inducing thermal and moisture variations throughout the structure's thickness. Thus, considering combined effects of temperature and humidity, termed hygrothermal, in deformation and stress analysis of thin-walled structures has been extensively studied by many researchers [1-2]. Moreover, Hygrothermal analysis of buildings for developing energy-efficient structures is important [3-4]. Determining moisture and temperature variations is a necessary step in hygrothermal structural analysis. Hence, significant research has been carried out in modeling heat and moisture transfer in different materials and geometries [5-6]. Heat and moisture diffusions are commonly recognized as coupled phenomena. Specifically, the rate of moisture diffusion, known as diffusivity, is temperature-dependent for most of materials. Herein, for brevity, researches on moisture diffusion within considering non-isothermal effects are reviewed.

Heide-Jørgensen et al. [7], conducted experiments to study moisture diffusion in a neat epoxy material and woven glass-epoxy composite layers at three different temperatures. They examined uniform temperature increases and found very good agreement between theoretical diffusion coefficients and experimental data, employing the Arrhenius law to model the relationship between diffusivity and temperature. Chapman [8] experimentally studied water diffusion in concrete at elevated and non-uniform temperatures in his Ph.D. thesis. Galbraith et al. [9] tested non-isothermal moisture diffusion in porous building materials. Peuhkuri et al. [10] experimentally measured non-isothermal water vapor transmission in porous building materials. They reported some indications that the temperature gradient drives moisture from warm to cold. Bai et al. [11] developed an analytical model that can estimate the moisture and ionic transfers in concrete driven by temperature gradient. Jervell et al. [12] clarified the role of thermal diffusion on the migration of water in humid air through porous insulation materials. Hancox [13] studied the drying of cement paste under the influence of temperature. He showed after the capillary process, diffusion effects predominate and moisture loss can be described adequately by simple diffusion theory. Ounaies and Ben Daly [14] measured diffusion coefficient of polyester/glass fiber composite in sea and distilled water.

Based on the above literature survey, the investigation of temperature gradients on moisture diffusion has recently gained attention from researchers. However, most of these studies used experimental methods, whose intrinsic complexity prevents substantial conclusions. Thus, in this work, we used a simple theoretical method to investigate the effects of non-uniform temperature distribution on 1-D moisture diffusion. The temperature distribution throughout the thickness is derived

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by solving Fourier's steady-state heat conduction equation for plate, cylindrical and spherical shells. Next, the Arrhenius-type relationship is utilized to incorporate the temperature dependency of diffusivity into Fick's diffusion equation. Because of non-uniform temperature variation, the moisture diffusion coefficient varies along the thickness direction. Subsequently, the moisture profile through the thickness is obtained by analytical solution of Fick's diffusion equation.

2. Formulation

In this section, the steady-state 1-D Fick's diffusion equation is derived and solved analytically. The Fick's diffusion equation may be written as [6]:

$$\frac{\partial C}{\partial t} = \nabla.(D\,\nabla C) \tag{1}$$

Where C is the moisture concentration, t is time, D is the diffusion coefficient or diffusivity and ∇ is used for Del operator. In the case of Isotropic materials, D is a scalar and in general is the materials property and dependent on the temperature and moisture. In this study, moisture dependency of D is not considered. In many applications, especially in thin-walled structures, moisture passes through the thickness, allowing the one-dimensional (1-D) form of Eq. (1) to be used under appropriate moisture boundary conditions. For plates, cylindrical and spherical shells, shown schematically in Fig. 1, the 1-D steady state diffusion equation simplified to:

Plate: $$\frac{\partial}{\partial z}\left(D\frac{\partial C}{\partial z}\right) = 0, \qquad a < z < a+h \tag{2}$$

Cylindrical shell: $$\frac{1}{r}\frac{\partial}{\partial r}\left(rD\frac{\partial C}{\partial r}\right) = 0, \qquad a < r < a+h \tag{3}$$

Spherical shell: $$\frac{1}{r^2}\frac{\partial}{\partial r}\left(Dr^2\frac{\partial C}{\partial r}\right) = 0, \qquad a < r < a+h \tag{4}$$

Where r shows the radial direction in cylindrical and spherical shells and h is the thickness of the structure as shown in Fig. 1. In the case of uniform temperature distribution, diffusivity D is constant and are cancelled in Eqs (2-4).

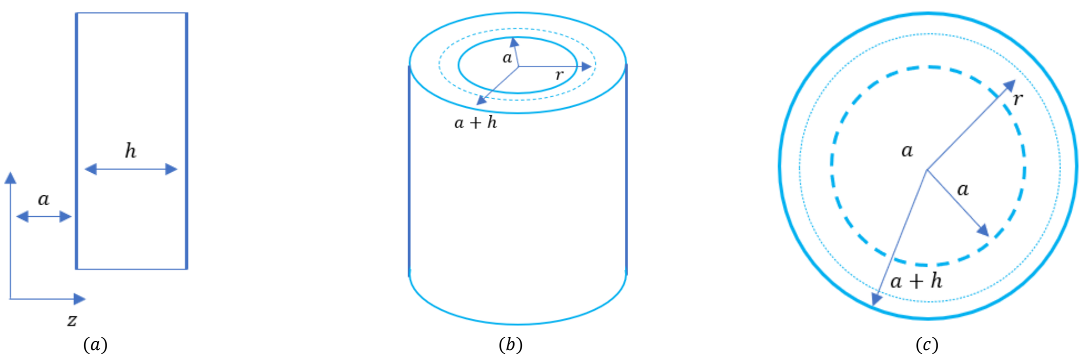


Fig. 1. Schematic geometry of structure, (a) plate, (b) cylindrical shell and (c) spherical shell

Here, Eqs. (2-4) are subject to prescribed moisture concentrations at the internal and external surfaces, determining the boundary conditions as follows:

$$C(a) = c_i, \qquad C(a+h) = c_o \tag{5}$$

The Arrhenius law is widely used to model temperature dependency of many parameters such as permeability, diffusion, cracking and creep [15]. It has also been demonstrated to work for moisture diffusion in polymers [16], which used as matrix in many composite structures, and for water diffusion and permeability in Portland cement paste [8, 17]. Based on Arrhenius model, temperature dependency of diffusion coefficient is:

$$D = D_0 \exp\left(-\frac{R}{T}\right) \tag{6}$$

Where $exp(\dots)$ is exponential function, T is temperature and D_0, R are materials constants which should be determined experimentally. The values of D_0, R for some composites in distilled and salt water, as well as humid air, are reported in [18]. The value of D_0 controls moisture saturation time and has no effects on steady state moisture distribution. The analytical solution of Eqs (2-4) subjected to boundary conditions in Eq. (5), are:

Plate: $$C(z) = \frac{c_o - c_i}{\int_a^{a+h} D dz}\left(\int_a^z D\,dz\right) + c_i, \qquad a < z < a+h \tag{7}$$

Cylindrical shell: $$C(r) = \frac{c_o - c_i}{\int_a^{a+h}\frac{dr}{rD}}\left(\int_a^r \frac{dr}{rD}\right) + c_i, \qquad a < r < a+h \tag{8}$$

Spherical shell: $$C(r) = \frac{c_o - c_i}{\int_a^{a+h}\frac{dr}{r^2 D}}\left(\int_a^r \frac{dr}{r^2 D}\right) + c_i, \qquad a < r < a+h \tag{9}$$

According to Eq. (6), for finding moisture concentration by Eqs (7-9), it's necessary to determine steady state temperature distribution through the thickness of the structure. As is well-known, there are a similarity between Fourier's heat conduction equation and Fick's moisture diffusion equation [6, 19]. By Substituting T instead C and κ, thermal conduction coefficient instead D in Eqs (2-4), steady state Fourier's equation may be obtained. Because κ is constant, solving the resulted equations are straightforward and temperature distribution in plates, cylindrical and spherical shells are obtained as:

Plate: $$T(z) = \frac{T_o - T_i}{h}(z - a) + T_i \tag{10}$$

Cylindrical shell: $$T(r) = \frac{T_i \ln(\frac{a+h}{r}) + T_o \ln(\frac{r}{a})}{\ln(\frac{a+h}{a})} \tag{11}$$

Spherical shell: $$T(r) = \frac{aT_i(a+h-r) + (a+h)T_o(r-a)}{rh} \tag{12}$$

Where T_i, T_o are prescribed temperatures on the inner and outer surfaces of the structure, respectively. According to Eqs (10-12), the temperature varies linearly in the plate but varies nonlinearly through the thickness of cylindrical and spherical shells. By substituting the temperature from Eqs (10-12) into Eq. (6) and the resulting equation into Eqs (7-9), the moisture concentration C can be found analytically. The obtained integrals can be expressed as exponential integral function with definition $Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t}\, dt$. However, the results are too lengthy to report here.

The average moisture absorbed by the structure or the average moisture content can be computed by:

$$m = \frac{\int_V C\, dV}{V} \tag{13}$$

Where V is the volume of the structure.

3. Discussion and Results

In this section, the equations for moisture distribution in plates, cylindrical, and spherical shells, Eqs (7-9) are solved numerically to investigate the effects of non-uniform temperature on the moisture profile through the thickness. In the following examples, the graphite/epoxy composite T300/5208 is considered, with the parameter R in Eq. (6) being 6750 for distilled water and 5912 for saturated salt water [18, 20]. The geometric parameters of the structures are assumed as $a = 1, h = 1$. In this case, the inner temperature $T_i = 293\ K$ and the outer temperature $T_o = 343\ K$ are considered. Using Eqs. (10-12), the temperature profiles through the thickness of the structures are shown in Fig. 2.

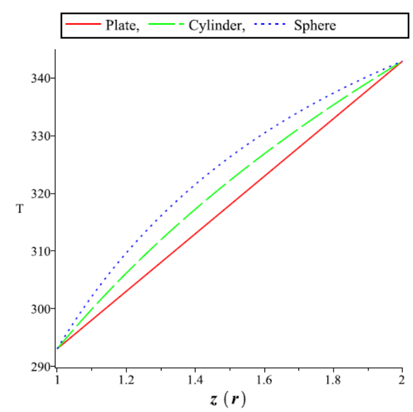
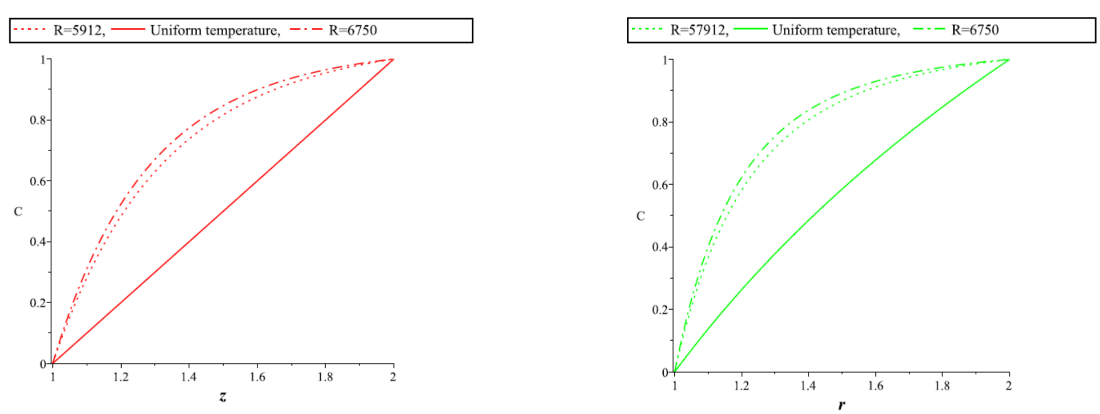
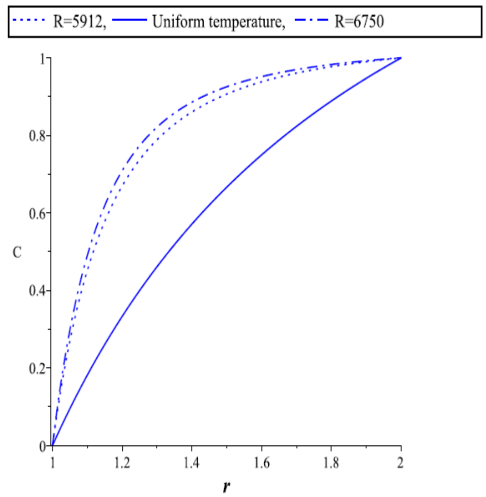


Fig. 2. Temperature profile through the thickness, $T_i = 293\ K, T_o = 343\ K$

As shown in Fig. 2, the temperature varies linearly in the plate, and as the thickness of the cylindrical and spherical shells decreases, their temperature profiles approach the linear case of the plate. In the Fig. 3, the resulted moisture distribution in the structures by assuming $c_i = 0, c_o = 1$, are plotted.



(a) (b)



(c)

Fig. 3. Moisture profile through the thickness for (a) plate, (b) cylindrical shell and (c) spherical shell

The moisture profile under uniform temperature in Fig. 3, where the inner and outer surfaces of the structure have the same temperature, does not depend on the temperature value. As shown in Fig. 3, the value of parameter R has a minor effect, but the impact of non-uniform temperature on moisture distribution is significant. The trend of moisture variation is similar in all three geometries. It is noteworthy that the un-uniform temperature $T_i = 293\ K, T_o = 343\ K$ cause significant higher steady state moisture concentration compared to uniform temperature, e.g., $T_i = T_o = 343\ K$.

Moisture content is an important physical parameter in the hygrothermal analysis. To investigate effects of non-uniform temperature on the moisture content of the structures, the average moisture absorbed from Eq. (13) are computed for the various outer surface temperature $T_o = 20 - 95\ ℃$ and plotted in Fig. 4. The inner temperature is kept as $T_i = 20\ ℃$.

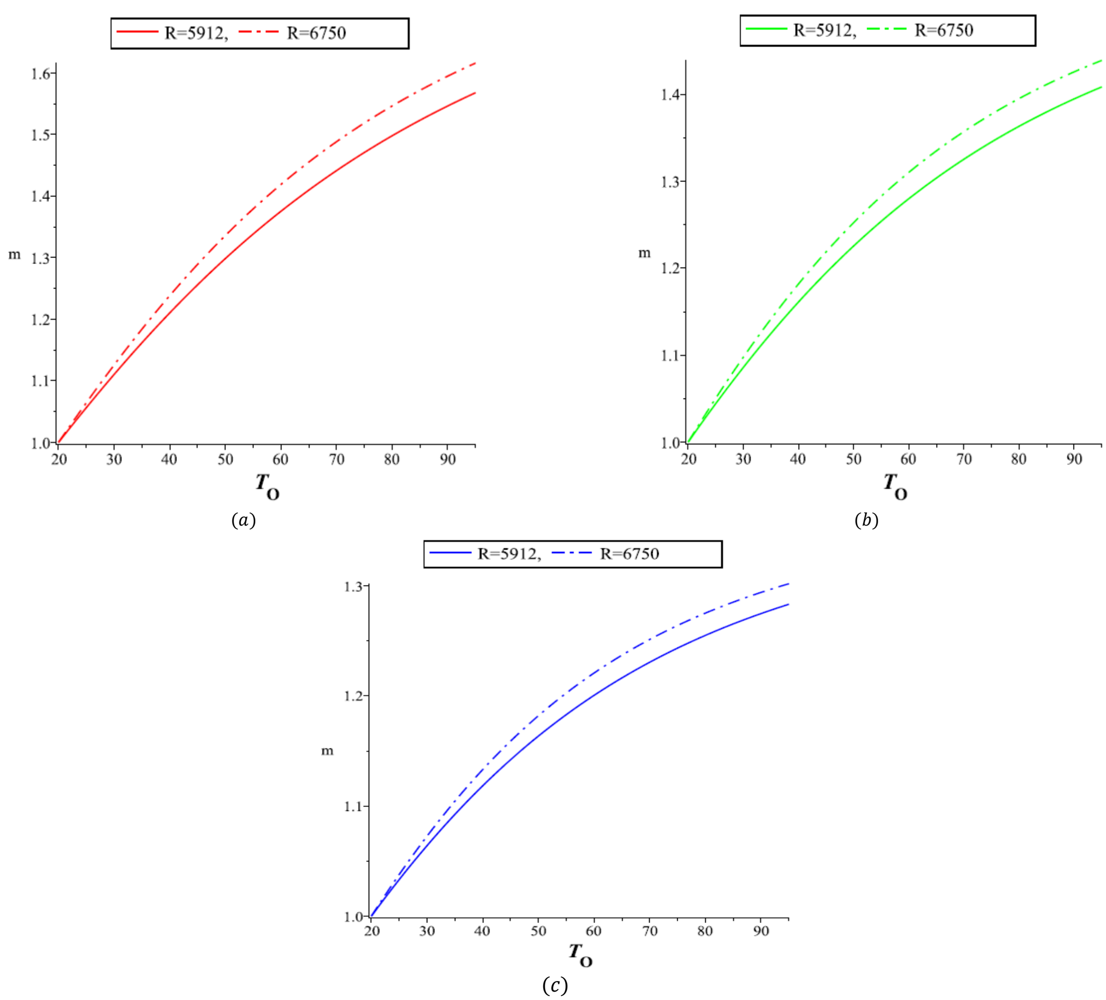


Fig. 4. Moisture content versus outer surface temperature for (a) plate, (b) cylindrical shell and (c) spherical shell

The average moisture content under uniform temperature distribution for all three structures are $m = 1$. As shown in Fig. 4, a higher temperature on the outer surface, with a moisture concentration $c_o = 1$, increases m by up to 60% at the $T_o = 95\ ^oC$ for the plate. The increase in moisture content for cylindrical and spherical shells are approximately 45% and 30% , respectively, at the $T_o = 95\ ^oC$ compared to uniform temperature. Moreover, different values of parameter R in Fig.4, indicates that the moisture content of the structures under contact with distilled water is larger than saturated salt water.

To investigate the effects of asymmetry in thermal and moisture boundary conditions, the inner and outer surfaces of the structures are set to $T_i = 343\ K, T_o = 293\ K$ and $c_i = 0, c_o = 1$. In other words, we assume the dry surface is warmer. The material and geometric parameters of the structure remain the same as in the previous example. The temperature profiles through the thickness of the structures are shown in Fig. 5.

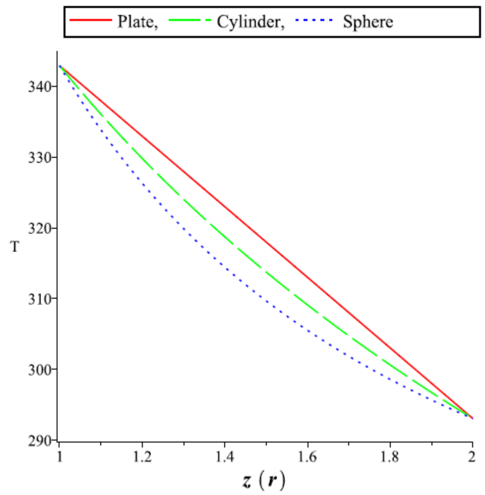
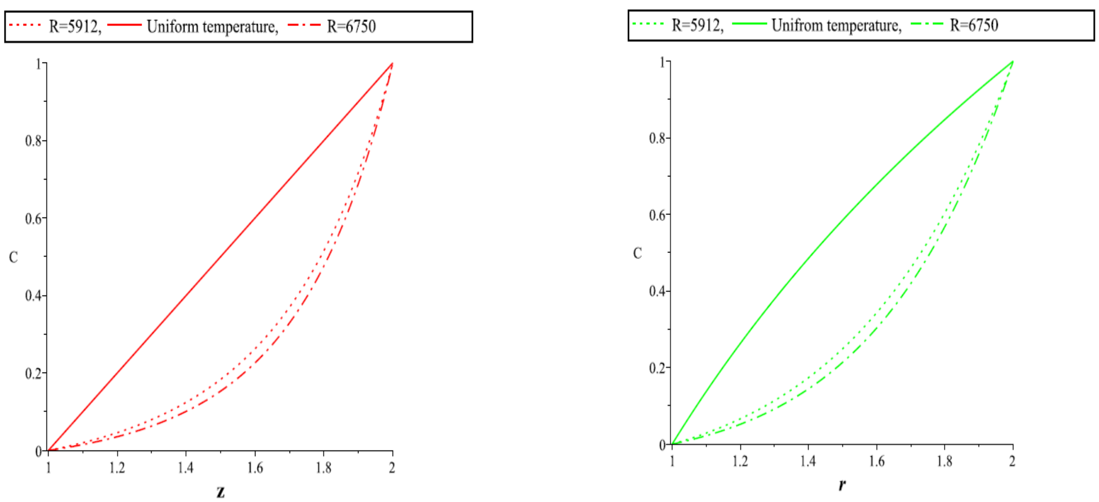
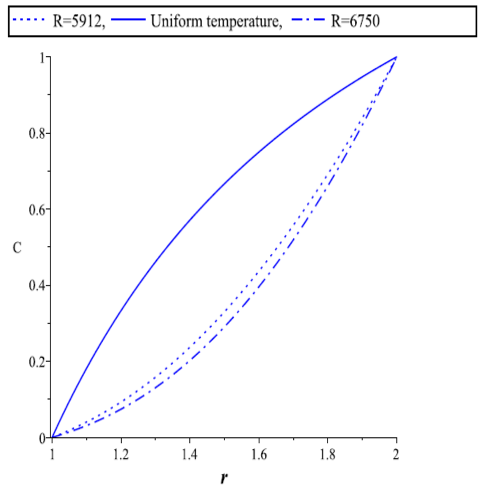


Fig. 5. Temperature profile through the thickness $T_i = 343\ K, T_o = 293\ K$

In the Fig. 6, the moisture distribution in the structures is depicted. As expected, the moisture profiles under uniform temperature are the same in Figs 3 and 6.





(c)

Fig. 6. Moisture profile through the thickness for (a) plate, (b) cylindrical shell and (c) spherical shell

Comparing Figs. 3 and 6 shows that the moisture concentration is considerably lower when the dry surface is hotter than when the moist surface is hotter. The average moisture content in the plate, are displayed in Fig. 7, which shows 60% decrease in moisture content compared to uniform temperature distribution at the $T_i = 95$ °C.

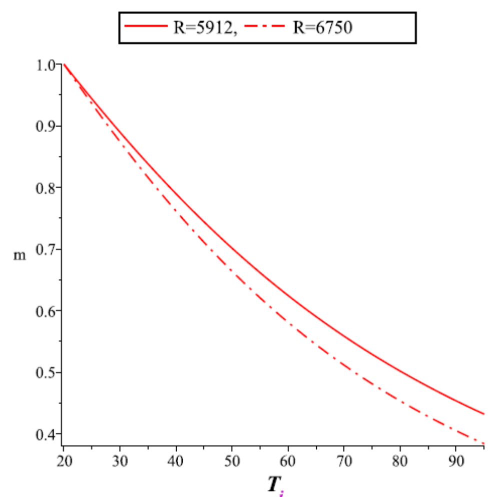


Fig. 7. Moisture content versus inner surface temperature for plate, $T_o = 293\ K$

4. Conclusion

In this study, steady state moisture profile through thickness of plates, cylindrical and spherical shells was investigated. Different temperatures on the internal and external surfaces of the structures cause variable diffusivity which modeled by Arrhenius-type relationship. The obtained Fick's diffusion equations were solved to find moisture distributions. The dependency of steady state moisture profile to non-uniform temperature at the structures was demonstrated. It has been shown that the moisture content is significantly higher when the direction of moisture diffusion and temperature gradient are the same, compared to the uniform temperature condition. The greatest increase was observed in the plate, while the smallest increase was seen in the spherical shell. When the drier surface is warmer, the situation reverses, and the moisture content shows a significant decrease compared to the uniform temperature condition.

Author Contributions

The author conceived and designed the study, conducted the research and numerical analyses, analyzed and interpreted the results, and wrote and approved the final version of the manuscript.

Conflict of Interest

The author declared no potential conflicts of interest concerning the research, authorship, and publication of this article.

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Data Availability Statements

The datasets generated and/or analyzed during the current study are available from the corresponding author on reasonable request.

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